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Four-Node Quadrilateral Shell Element MITC4

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Abstract. Four-node quadrilateral element MITC4 applicable to both thick and thin shells is presented. The element formulation starts from three-dimensional continuum description degenerated to shell behavior. Shear locking, which is common problem in analysis of thin shells, is overcome by the use of MITC (Mixed Interpolation of Tensorial Components) approach. Element has been implemented into finite element code OOFEM and its performance is demonstrated on Scordelis-Lo shell, a benchmark problem frequently used in the evaluation of shell elements.

Introduction

Shell structures are widely used in structural engineering for their load-carrying efficiency. The finite element method appears to be the most powerful tool in analysis of shell structures, however the accuracy of the method is significantly influenced by the choice of the particular element. Many shell elements have been developed over the years. Existing elements can be divided into two main groups. One group consists of elements based on some particular shell theory, the other group of the elements is based on three-dimensional analysis degenerated to shell behavior.

Presented quadrilateral element has been proposed by Dvorkin and Bathe [1] and is based on the second approach, therefore it is independent of any particular shell theory. The authors aimed to formulate element applicable to both thick and thin shells of arbitrary geometries with all degrees of freedom concentrated to the vertices of the element. However high-order elements with 9 and 16 nodes have been already successfully employed by Bathe and Bolourchi [2], this 4-node element suffers from the deficiency of shear locking. To prevent the element from the shear locking the use of MITC approach has been proposed.

Element Formulation

The element is shown in Fig. 1 and its geometry is described using

$$x_i = \sum_{k=1}^{4} h_k x_i^k + \frac{r_3}{2} \sum_{k=1}^{4} a_k h_k V_{ni}^k, \tag{1}$$

where $h_k(r_1, r_2)$ are the two-dimensional interpolation functions corresponding to node k, r_i are the natural coordinates, x_i are the Cartesian coordinates of any point in the element, x_i^k are the Cartesian coordinates of node k, V_{ni}^k are the components of director vector at node k and a_k is the thickness of structure measured along the director vector. The displacement of any point in the element can be described using

$$u_i = \sum_{k=1}^{4} h_k u_i^k + \frac{r_3}{2} \sum_{k=1}^{4} a_k h_k (-V_{2i}^k \alpha_k + V_{1i}^k \beta_k), \tag{2}$$

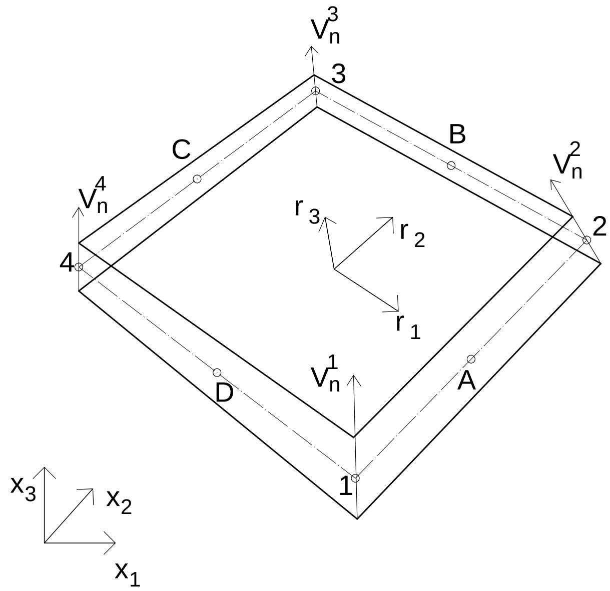

Fig. 1: Geometry of quadrilateral shell element MITC4.

where u_i are the displacements in direction of Cartesian coordinates and α_k and β_k are the rotations of director vector at node k around V_1^k and V_2^k, where

$$V_1^k = \frac{\boldsymbol{e}_2 \times V_n^k}{\|\boldsymbol{e}_2 \times V_n^k\|}, \tag{3}$$

$$V_2^k = V_n^k \times V_1^k. \tag{4}$$

Vector $\boldsymbol{e}_2$ is the basis vector of Cartesian coordinate system. Considering linear interpolation of displacement field, we introduce the vector of unknown displacements

$$\boldsymbol{r}_e = \{u_1^1, u_2^1, u_3^1, \alpha_1, \beta_1, u_1^2, u_2^2, u_3^2, \alpha_2, \beta_2, u_1^3, u_2^3, u_3^3, \alpha_3, \beta_3, u_1^4, u_2^4, u_3^4, \alpha_4, \beta_4\}^T. \tag{5}$$

The biggest drawback of this formulation of the element is that the element locks when the thickness of shell is small. Using the interpolation (2), non-zero transverse shear strains are obtained even when the structure is subjected to the constant bending moment. The remedy is to construct an assumed transverse strain, such that

$$\begin{aligned} \tilde{\varepsilon}_{13} &= \tfrac{1}{2}(1 + r_2)\tilde{\varepsilon}_{13}^A + \tfrac{1}{2}(1 - r_2)\tilde{\varepsilon}_{13}^C, \\ \tilde{\varepsilon}_{23} &= \tfrac{1}{2}(1 + r_1)\tilde{\varepsilon}_{23}^D + \tfrac{1}{2}(1 - r_1)\tilde{\varepsilon}_{23}^B, \end{aligned} \tag{6}$$

where $\sim$ over the quantities emphasizes the formulation in convected coordinate system and components $\tilde{\varepsilon}_{13}^A$, $\tilde{\varepsilon}_{23}^B$, $\tilde{\varepsilon}_{13}^C$ and $\tilde{\varepsilon}_{23}^D$ are equal to the values of corresponding strains in the middle of element edges calculated from displacements. This assumed transversal strain $\tilde{\varepsilon}_{13}$ is constant along r_1 direction and linear along r_2 direction.

To transform formulas for shear strain components to Cartesian coordinate system, it is convenient to use covariant and contravariant bases in convected coordinate system of r_1, r_2 and r_3. For the element with midsurface in plane x, y the covariant basis vectors $\boldsymbol{g}_i$ $(i = 1, 2, 3)$ are given by

$$\boldsymbol{g}_i = \frac{\partial \boldsymbol{x}}{\partial r_i}. \tag{7}$$

Contravariant basis vectors $\boldsymbol{g}^j$ $(j = 1, 2, 3)$ can be then calculated using

$$\boldsymbol{g}_i \boldsymbol{g}^j = \delta_i^j, \tag{8}$$

where δ_i^j is Kronecker delta, $\delta_i^j = 1$ for $i = j$ and $\delta_i^j = 0$ otherwise. Components $\tilde{\varepsilon}_{ij}$ can be evaluated using

$$\tilde{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \boldsymbol{u}}{\partial r_i}\boldsymbol{g}_j + \frac{\partial \boldsymbol{u}}{\partial r_j}\boldsymbol{g}_i\right), \tag{9}$$

$$\tilde{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \boldsymbol{u}}{\partial r_i}\frac{\partial \boldsymbol{x}}{\partial r_j} + \frac{\partial \boldsymbol{u}}{\partial r_j}\frac{\partial \boldsymbol{x}}{\partial r_i}\right). \tag{10}$$

By substituting from (1) and (2) into equation (10) and evaluating at points A, B, C, D the relations for $\tilde{\varepsilon}_{13}^A$, $\tilde{\varepsilon}_{23}^B$, $\tilde{\varepsilon}_{13}^C$ and $\tilde{\varepsilon}_{23}^D$ can be obtained

$$\begin{aligned}
\tilde{\varepsilon}_{13}^A = \; & \tfrac{1}{8}\left[(\boldsymbol{u}^1 - \boldsymbol{u}^2)\cdot\tfrac{1}{2}(a_1 V_n^1 + a_2 V_n^2) + \right.\\
& \left.(\boldsymbol{x}^1 - \boldsymbol{x}^2)\cdot\tfrac{1}{2}(a_1(-V_2^1\alpha_1 + V_1^1\beta_1) + a_2(-V_2^2\alpha_2 + V_1^2\beta_2))\right],\\
\tilde{\varepsilon}_{13}^C = \; & \tfrac{1}{8}\left[(\boldsymbol{u}^4 - \boldsymbol{u}^3)\cdot\tfrac{1}{2}(a_3 V_n^3 + a_4 V_n^4) + \right.\\
& \left.(\boldsymbol{x}^4 - \boldsymbol{x}^3)\cdot\tfrac{1}{2}(a_3(-V_2^3\alpha_3 + V_1^3\beta_3) + a_4(-V_2^4\alpha_4 + V_1^4\beta_4))\right],\\
\tilde{\varepsilon}_{23}^B = \; & \tfrac{1}{8}\left[(\boldsymbol{u}^1 - \boldsymbol{u}^4)\cdot\tfrac{1}{2}(a_1 V_n^1 + a_4 V_n^4) + \right.\\
& \left.(\boldsymbol{x}^1 - \boldsymbol{x}^4)\cdot\tfrac{1}{2}(a_1(-V_2^1\alpha_1 + V_1^1\beta_1) + a_4(-V_2^4\alpha_4 + V_1^4\beta_4))\right],\\
\tilde{\varepsilon}_{23}^D = \; & \tfrac{1}{8}\left[(\boldsymbol{u}^2 - \boldsymbol{u}^3)\cdot\tfrac{1}{2}(a_2 V_n^2 + a_3 V_n^3) + \right.\\
& \left.(\boldsymbol{x}^2 - \boldsymbol{x}^3)\cdot\tfrac{1}{2}(a_2(-V_2^2\alpha_2 + V_1^2\beta_2) + a_3(-V_2^3\alpha_3 + V_1^3\beta_3))\right].
\end{aligned} \tag{11}$$

Final equations for $\tilde{\varepsilon}_{13}$ and $\tilde{\varepsilon}_{23}$ are then derived by substituting (11) back into the equations (6). Transformation to the Cartesian coordinates is performed using

$$\tilde{\varepsilon}_{ij}\boldsymbol{g}^i\boldsymbol{g}^j = \varepsilon_{kl}\boldsymbol{e}_k\boldsymbol{e}_l, \tag{12}$$

where ε_{kl} are components of strain tensor in Cartesian coordinates with basis vectors $\boldsymbol{e}_k$ and $\boldsymbol{e}_l$. By considering the symmetry of strain tensor, the shear components of strain vector γ_{xz} and γ_{yz} are obtained

$$\begin{aligned}
\gamma_{xy} &= 2\tilde{\varepsilon}_{13}(\boldsymbol{g}^1\cdot\boldsymbol{e}_1)(\boldsymbol{g}^3\cdot\boldsymbol{e}_3) + 2\tilde{\varepsilon}_{23}(\boldsymbol{g}^2\cdot\boldsymbol{e}_1)(\boldsymbol{g}^3\cdot\boldsymbol{e}_3),\\
\gamma_{yz} &= 2\tilde{\varepsilon}_{13}(\boldsymbol{g}^1\cdot\boldsymbol{e}_2)(\boldsymbol{g}^3\cdot\boldsymbol{e}_3) + 2\tilde{\varepsilon}_{23}(\boldsymbol{g}^2\cdot\boldsymbol{e}_2)(\boldsymbol{g}^3\cdot\boldsymbol{e}_3)
\end{aligned} \tag{13}$$

Substituting from (6) and (11) into (13) the final formulas for transverse shear strains are derived. The remaining components of strain vector are calculated directly from the displacements.

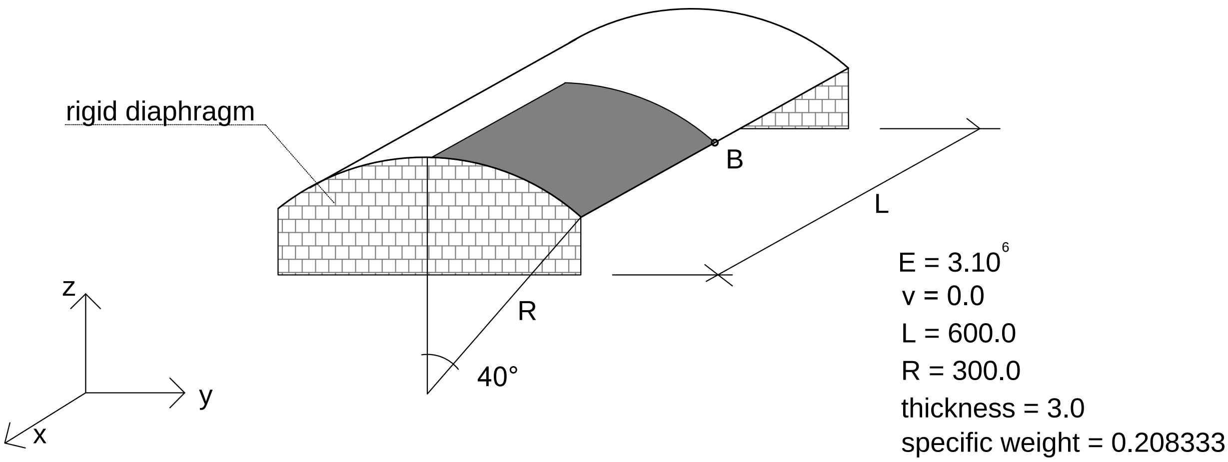

Fig. 2: Cylindrical Scordelis-Lo shell subjected to dead weight.

Stiffness matrix is evaluated using

$$\boldsymbol{K} = \int_V \boldsymbol{B}^T \boldsymbol{D} \boldsymbol{B} \, dV, \tag{14}$$

where $\boldsymbol{B}$ is a strain-displacement matrix resulting from derived formulas for strain components and $\boldsymbol{D}$ is a material matrix for three-dimensional problem degenerated to shell behaviour, where the condition of $\sigma_z = 0$ is enforced.

Scordelis-Lo Shell Problem

Element has been implemented into finite element code OOFEM [3, 4] and implementation has been verified using classical patch tests. It has been proven, that the patch tests are passed for the case of pure bending, pure shear and pure twist and also for three membrane stress states.

To test the element performance on more complex structure and to compare its results with other available elements and with the analytical solution the analysis of Scordelis-Lo Shell [1] is presented. The structure is loaded by its dead weight and is shown in Fig. 2. Due to the symmetry only one quarter of the structure has been analyzed. Deformed shape of the structure is shown in Fig. 3, obtained results are shown in Fig. 4. Solution obtained using the element composed of plane-stress element with rotational degrees of freedom and Discrete Kirchhoff Triangle plate element, is also shown for reference, labeled as RDKT.

Summary

Element for shells has been implemented into existing finite element code OOFEM and its performance has been verified. Shear locking, which can cause highly inaccurate results in analysis of thin shells, has been overcome by MITC approach. This approach seems to be sufficient as appropriate results were obtained while testing the element implementation. Numerical results has shown that the element is competitive with the performance of thin-plate elements. Possibility of use of MITC4 element for both thick and thin shells can be seen as advantage over the RDKT and other shell theory based elements, which are usually limited by thickness/span ratio to either thin or thick shells. The element shows very good convergence to the reference solution [1], even outperforming the planar shell element composed of thin Kirchhoff plate element and membrane element.

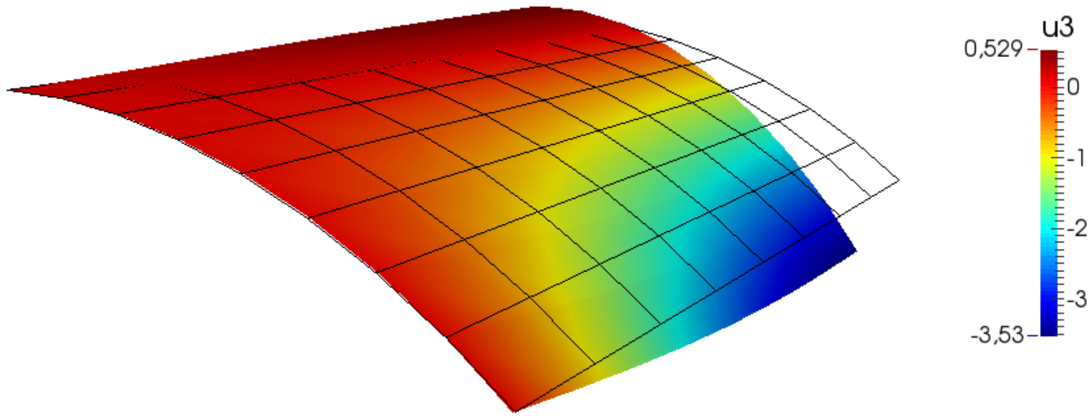

Fig. 3: Cylindrical Scordelis-Lo shell subjected to dead weight. Only one quarter of the structure has been analyzed, mesh of 8×8 elements and deformed shape are shown.

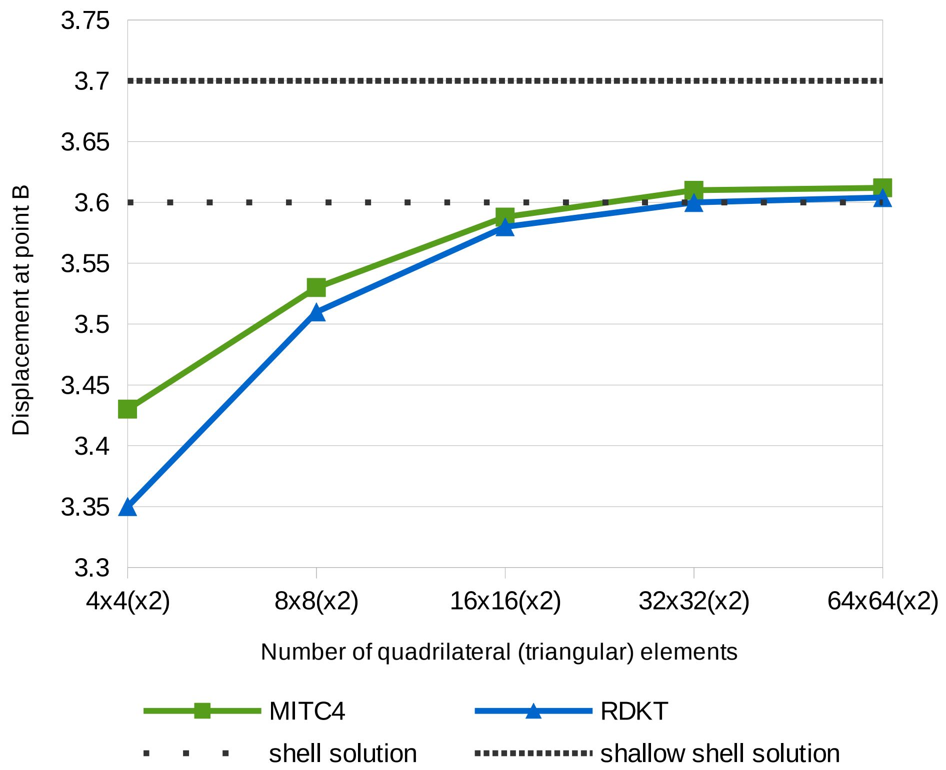

Fig. 4: Analysis of cylindrical Scordelis-Lo shell subjected to dead weight. Convergence of displacement at point B is studied, solutions obtained with MITC4 and RDKT elements are shown as well as analytical solutions.

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